LONG ISLAND
BOTANICAL SOCIETY

Mathematical Models
of Botanical Forms

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A number of years ago I figured out how to write a computer program that would draw a simple stick figure fractal tree on my first IBM PC. The idea that a simple five line function could produce a geometric object of such complexity was a revelation to me. Computer graphics has made it possible to model plant growth and even to invent exotic imaginary plants by observing the patterns in nature and compounding them.

A simple technique in mathematics called *recursion* can be implemented easily in a computer program. A recursive function is a function that calls itself, so a recursive function allows us to repeat a simple replacement rule over and over again, changing parameters at each stage. This is apparently what happens in nature; each seed of a plant contains all the instructions necessary to tell that plant when to branch, when to flower and when to slow its growth rate. Plant structures illustrate what fractal geometers call *self-similarity*. A branch of a tree looks like a small replica of the tree itself and even the vein structure of a leaf often looks like a two dimensional picture of the tree it came from. The technique of recursion allows us to generate patterns of enormous complexity by just specifying a few simple directions, and then letting the program take over. For example, suppose we start with a single line segment and a simple replacement rule which says: at each time stage, replace each line segment in the current figure by five new line segments as in Figure 1a. Figure 1b. shows what the figure looks like at several stages in its development. After several stages it bears some resemblance to a real

plant. If we add more parameters such as length, thickness and curvature of the line segments, we have a somewhat realistic looking plant. We can also use random numbers and probabilities in a program to simulate the effects of external forces such as the amount of light, temperature and nutrients a plant receives; then the branches of our computer-generated plant and the branching angles are not identical as they are in the "plant" in Figure 1.
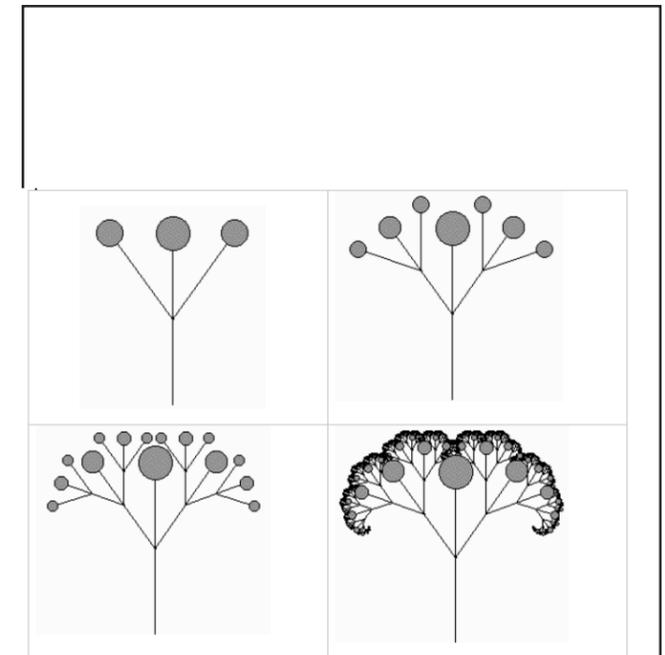

Figure 1. Dichasium, stages 1, 2, 3 and 10

American Chestnut (*Castanea dentata*)

Society News

Calverton Field Reprieve: The former Grumman Corporation facility site in Calverton was chosen by promoters of "Field Day": a mammoth three-day rock concert/camping festival with an expected 35,000 to 50,000 attendees. The event, planned earlier in the year, was to take place June 6 - June 8 with 36 rock bands scheduled.

   The Field Day Fest was to be held in the New York State Central Pine Barrens Preserve, within a state-designated Special Groundwater Protection Area, directly adjacent to the Core Preservation Area (the most ecologically sensitive part of the Pine Barrens. Directly adjacent to the festivities are globally rare Coastal Plain Ponds, managed by The Nature Conservancy, that contain endangered species like the Eastern Tiger Salamander. Nesting on the ground, in the grassy areas (where the concert was to take place) are rare grassland birds including Grasshopper Sparrows, a New York State listed "Species of Special Concern", Horned Larks, and Eastern Meadowlarks. During early June the nests are active with hatching eggs and newborn young which cannot yet fly.

   The Long Island Pine Barrens Society took the lead in preventing the event from taking place by initially filing an injunction action against the Town of Riverhead which was hosting the event. Ultimately, a town board meeting was held 48 hours before the gates were to open. Two hours of public comment were heard before a standing room only crowd of mostly environmentalists and residents opposed to the event. It was brought out at the meeting that Town Supervisor Robert Kozakiewicz of Riverhead had contacted a concert ticket holder through the website of the Field Day website's message board urging concert supporters to attend the Town Board meeting because opposers of the concert would certainly be there. The illustrious Dick Amper, who refused to be silenced by the town supervisor was ultimately escorted by police from the meeting. When the dust settled, the town board voted 5-0 to cancel the event.

Orient Point Disappointment: Debate over the allowance of boat landings on the long, narrow western portion of Orient Point State Park preserve continued between environmentalists and a small group of aggresive locals who wished to beach their boats on the dunes. This portion, which has National Landmark status (which unfortunatley has little protective or enforcement strength), hosts nesting Piping Plovers and a collection of rare plants, including one of only three locations on Long Island of *Sesuvium maritimum*. New York State Department of Parks settled the issue by granting the aggressive individuals their boat beaching rights opening the door for degradation of the site. This leaves the Piping Plovers, confined to fewer and fewer areas on Long Island, with one less secure nesting location. And the rare maritime plants, that thought they had a safe haven to grow within a nationally recognized site, with one less site solely devoted to the protection of natural history.

Another of my favorite ways to model botanical growth is a method called
L-Systems, or string rewriting. It was discovered by Aristid Lindenmayer, a Dutch biologist who hit upon the remarkable idea of using ideas from formal language theory to describe plant growth. In this model each geometric part of a plant is assigned a character; I use I to represent an internode, L a leaf, F a flower, etc. We use brackets and parenthesis to indicate branches. For example the string of characters I[I[L](L)IF]I(II(L)IF) IF might represent the (purely imaginary) plant in the first frame of Figure 7. We then use a set of rewriting rules or productions to successively replace each character by a string of characters at each stage in the development of the plant. Signals can be propagated through the strings to tell a plant when to flower, when to slow its growth rate, when to change from leaf creation to branch creation along an axis, etc. The book by Prusinkiewicz (Number 7 in References) is an excellent introduction to a mathematical theory of plant development. Figure 7 illusrates several stages in the development of an imaginary plant form using this method.

A fourth method of modeling plant growth and other branching phenomena uses a stochastic matrix of probabilities; the method is reminiscent of a Markov chain. Space considerations preclude a description here, but the interested reader can find more in Reference 2.

I use other mathematical techniques to model mountains and terrains, clouds and water. I then combine them into what I call "Mathscapes". Figure 8 shows two examples of these Mathscapes. These pictures contain artificial trees and plants modeled using some of the above methods.

On my web site at http:\\www.phoenix.liu.edu/ ~aburns/ can be found more examples of Mathscapes as well as some interactive applets that allow further exploration into String Rewriting and Real and Imaginary Inflorescences.

Scan Figure 7

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References:

1. Burns, Anne M., The Flowering of String Rewriting Systems, *College Mathematics Journal,* Vol 23 No. 3, (1992)

2. Burns, Anne M., Modeling Trees With a Stochastic Matrix, *College Mathematics Journal,* Vol 29 No. 3, (1998)

3. Burns, Anne M., Making Mountains from a Sum of Molehills, *College Mathematics Journal,* Vol 26 No. 1, (1995)

4. Burns, Anne M., Real and Imaginary Inflorescences, *UMAP*, the Journal of Undergraduate Mathematics and Its Applications, Volume 21 Number 4, 2000

5. de Reffye, Philippe, et al, Plant Models Faithful to Botanical Structure and Development, *Computer Graphics*, Volume 22, Number 4, August, 1998

6. Porter, C. L, *Taxonomy of Flowering Plants*, San Francisco, CA; W.H. Freeman

7. Prusinkiewicz, Premyslaw, and Aristid Lindenmayer, *The Algorithmic Beauty of Plants: the Virtual Laboratory,* Springer-Verlag, New York, 1990

Letters to the Editor

Dear Barbara (Conolly) and all my friends at the Long Island Botanical Society,
This is botanical and ornithological paradise. I have gotten affiliated with the California Native Plant Society. There is so much to do here and never enough time. Stanislaus National Forest is only 8 miles away and the rangers have been very helpful. I took a 4-wheel drive into a very historic and scenic part. It was bumpy, narrow, dusty, but oh, so wonderful. What a learning experience.
I bought a 2003 Suburban Forester (all wheel drive). I need it on these winding mountain roads. We are at 4000' (foothills of the Sierra Nevada). Magnificent scenery. A mountain lion crossed the road about 3 miles from here.
I took a drive to a rather remote, beautiful valley of the Tuolumne River which was dammed to create a reservoir for San Francisco. When I was alone on the bridge, a bear and her cub approached me. I slowly retreated to the car and she and her cub disappeared into the mountainside.
Bill Redshaw visited for ten days. He enjoyed the comments which you all made on the card. Thanks so much for it. My daughter-in-law, Carole, Bill and I took a trip to the Calaveros Big Trees - Giant Sequoias. The largest tree has a base diameter of 30 feet and at 48 feet above the ground it is still 16 feet in diameter. The way to appreciate their size is to lie on the ground and look upward so that's how we did it. We saw *Taxus brevifolia*, *Cornus nuttallii* (6 bracts), and *Corylus cornuta*.
I wish you could come for a visit. I think you would love the botanizing. I'd pick you up at Sacramento Airport (Southwest Airlines) and we'd be off.
Love,
Elsa L'Hommedieu

I am writing to ask you to include a notice about the upcoming New York State Outdoor Education Association Conference. [Villa Roma Conference Center, Callicoon, NY; Oct. 23-26, 2003; Information: George Steele, 518-842-0501 nysoea@aol.com; Registration: 607-674-4017, conmecho@norwich.net; Lodging: 800-727-8455]
Thank you,
Daniel Kriesberg

Plant Sightings

Juniper: Rich Kelly came up with a find of *Juniperus communis* in Stilwell Woods, Syosset, the only known sighting in Nassau County.

Grape Fern: Rich Kelly also reported *Botrychium dissectum v. obliquum* in Caumsett State Park right under the water tower where LIBS members on a field trip had looked for it last year.

Storksbill: Barbara Conolly repoorted a profusion of Storksbill (*Erodium cicutarium*) in early April on the grass edges of the highway approaching Jones Beach West End 2. Rich Kelly and Skip Blanchard have seen it also in other places, such as the west end of Fire Island in early May. John Potente has also seen it in May along Veterans Memorial Highway in Hauppauge.

Ipecae Spurge: Skip and Rich have recently seen Ipecac Spurge (*Euphorbia ipecacuanhae*) in flower at Caleb Smith State Park. They had reported it begetatively last fall in the same location. Ray Welch volunteered that he had seen the Ipecac Spurge on Suffolk Community College Campus 8 or 9 years ago and that there were 20 green morph and 25 red morph specimens.

American Chestnut: Margaret Conover relayed that the American Chestnut tree (*Castanea dentata*) in her yard has succumbed to blight during last August's drought and this year has shown no signs of bud break. The trunk now shows the classic swollen and split bark. John Potente had this tree inventoried since 1996 as one of the best examples of a surviving American Chestnut tree on Long Island. It was well over twenty five feet high, flowered copiously and had a trunk girth of about ten inches dbh. (The tree is pictured in the closing photograph of the article: "Retrieving the American Chestnut" in LIBS Vol.13, No.1)

New Members

Dale Bente   Islip, NY
Lynda Cosgrove-Scollo   Islandia, NY
Janis Enderle   Dix Hills, NY
Marlies Kneis   Port Jefferson Station, NY
Rosemary Schuylenburg   Rocky Point, NY
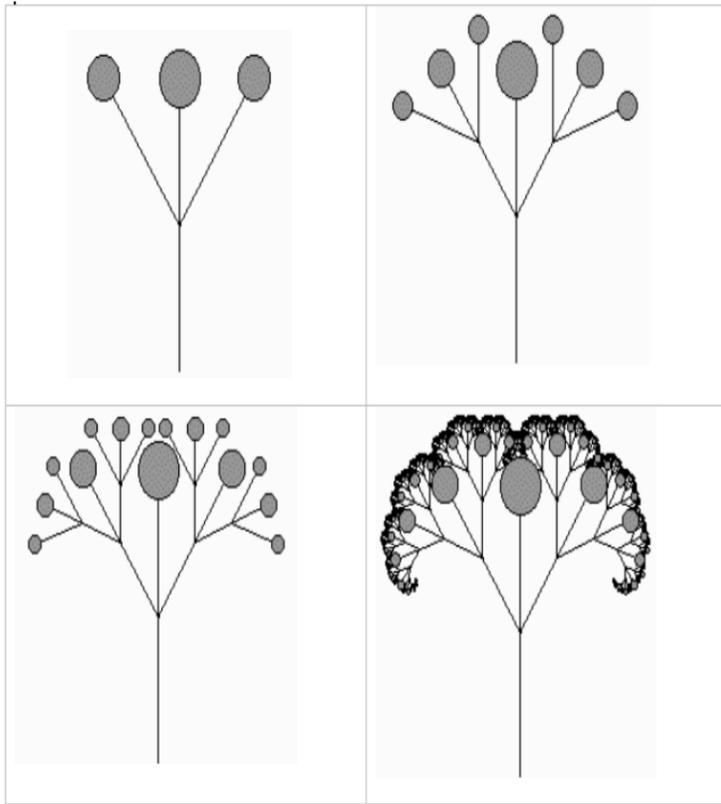**New Life Member:** Tom Fiore   New York, NY

Figure 1. Dichasium, stages 1, 2, 3 and 10

Note: This figure 1 (from Anne's disc) does not correspond to the printout Figure 1 she gave me.
Use the print-out.

When implementing recursion on a computer we have to tell the function when to stop drawing; so one of the parameters of the recursive function will be the number of stages to draw. If we let n be a positive integer greater than 1 then an algorithm for the stage n plant in Figure 1 might look something like this:

If the stage = 0 then draw a line segment at angle 90 degrees from the horizontal
Otherwise:
    Draw a plant at stage n-1 at the current angle
    Draw a plant at stage n-1 at the current angle + 30 degrees
    Draw a plant at stage n-1 at the current angle
    Draw a plant at stage n-1 at the current angle - 30 degrees
    Draw a plant at stage n-1 at the current angle
The reader should try tracing through this algorithm starting with n = 3 and using Figure 1 as a guide.

A colleague from the Biology Department, Dr. Orland Blanchard, introduced me to C.L. Porter's *Taxonomy of Flowering Plants*. In this book are diagrams of the common inflorescences or compound flowering structures: stick figures with small disks representing the individual flowers. The diagrams of the compound inflorescences looked similar to my computer graphic stick figures; I realized that using recursion I could not only reproduce them but could carry out the recursion to any desired number of stages. I start, as Porter did, with a simple initial figure, a line segment topped with a disk. Figure 2 shows some of the common inflorescences. For each inflorescence a simple mathematical rule for compounding can be detected by looking at Porter's diagrams. Figure 3 shows the compounding of some of the inflorescences in Figure 2. Then I decided to have some fun. In programming one can form a recursive chain, where one recursive function calls a second recursive function; it can call another and so on, and the last one calls the first. This way I was able to invent some imaginary inflorescences by using more than one of the inflorescence algorithms in the same program. Figure 4 shows some imaginary inflorescences

Another model for plant growth is a simplification of an idea developed by Philippe de Reffye. For each node in a plant we assign three probabilities: p1, the probability that a node branches or ramifies, p2, the probability that the node dies, or becomes a flower and then dies, and p3, the probability that the node "sleeps" or does nothing until the next time period. Since p1, p2 and p3 are the probabilities of mutually exclusive events and their union is a certain event, we require that p1 + p2 + p3 = 1. At each time stage in the development of the plant we examine each node that is alive, generate a pseudo random number r between 0 and 1, and use that number to determine whether that node ramifies, dies or sleeps; if $r < p_1$ the node ramifies, if $p1 \leq r \leq p2$ the node dies, or flowers and dies, if $p2 \leq r$ the node sleeps. I specify a *ramification number* for a particular model, which is the number of branches emanating from a node for a plant. From examining some trees in my yard, the ramification number for an oak tree is 2, for a maple tree is 3 and for a cork tree is 5. (For some plants it is not always the same number at a branching node.)

In this model we recursively define the *order* of a node to be 0 if it is the single original node, and *n+1* if its parent node has order *n*. This definition is in keeping with standard botanical definition. In a computer model of a tree. In the simplest model the information stored for each live node is its x, y coordinates, its current angle of growth and its order. At each time unit we traverse the list. For each node we determine its next state using the three probabilities. Let *r* be its ramification number. If it ramifies we draw *r* internodes from it with each internode ending in a new live node. We enter these *r* new nodes into the list and delete the original node. If it dies, we delete it from the list (perhaps drawing a flower in its place first). If it sleeps we leave it in the list and go on to the next node. We can make use of the order of each node in several ways: to determine the width and length of the branches emanating from it, to determine the branching angle, but perhaps most importantly to determine the three probabilities, p_1, p_2, and p_3. If our three probabilities are variables we can make them functions of the order of a node. We would like a node in the early stage of a tree's growth to have a high probability of ramifying and a low probability of dying. As the tree ages we want a node to have a higher probability of dying. In real life a tree may grow for thousands of years, but on our computer screen we would like our tree to stop growing at some point.

In Figure 5 we show three "trees" using a ramification number of 2. The exact same program drew the three trees; since we use probabilities and random numbers we make it look as if the three trees come from the same family, but are not identical.
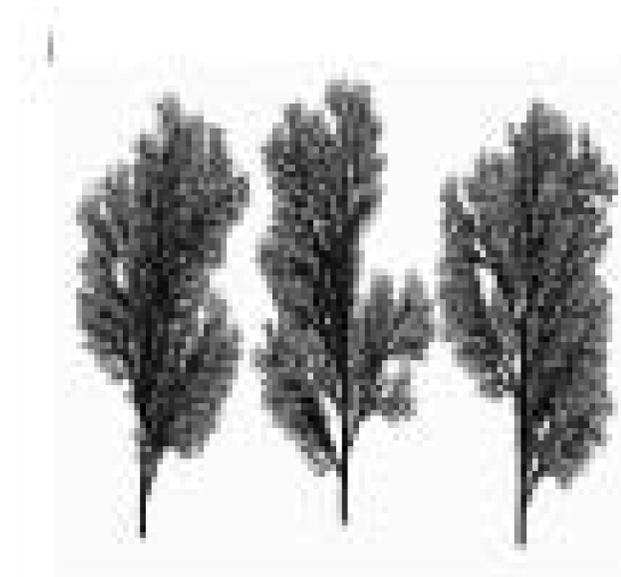

Figure 5.

Three "trees" using a ramification number of 2. The exact same program drew the three trees, using probabilities and random numbers we can make it look as if the three trees come from the same family, but are not identical.
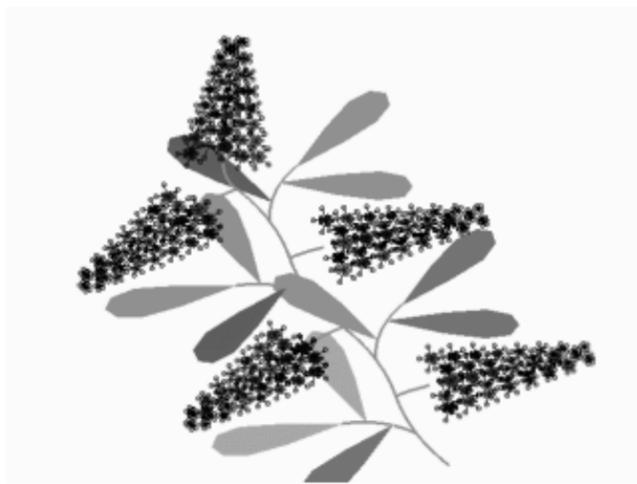
Scan in Figure 2

Scan Figure 3

Figure 4. Some imaginary inflorescences

Anne Burns, 01/03

Anne Burns, 2002